Use of graphs eases transformer selection for linear supplies

Engineers generally use simple rules of thumb when selecting transformers for linear power supplies. These rules of thumb aren't universally applicable, however, and blindly using them may cause you to select a less-than-optimal transformer—and thus a less-than-optimal supply.

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If you're designing a linear power supply that will use a transformer operating at full rated load with a load-regulation factor of 0.9, traditional rules of thumb for selecting the transformer will suffice. For other applications, these rules won't necessarily be sufficient. You can account for varying power-supply operating parameters for all operating conditions by expressing the equations in the **box**, "Circuit models yield design equations," in the form of easy-to-use graphs. These equations are derived from simple models of common power-supply topologies (**Fig 1**).

Modeling power supplies' behavior involves some simplifying assumptions. The models used to produce the graphs in this article assume that you can ignore the effects of temperature and mains-voltage variations; assume that diodes conduct abruptly, have a constant forward-voltage drop, and have a negligible series resistance; and assume that the filter capacitors have a negligible equivalent series resistance and such a large capacitance that the ripple voltage (the ac voltage

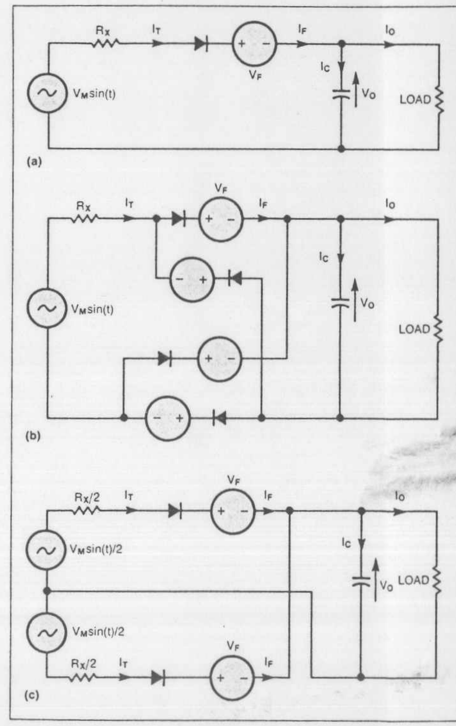

***Fig 1—For each linear-power-supply topology**—half-wave (**a**,) full-wave bridge (**b**,) and full-wave center-tap (**c**)—you can use **Figs 2** through **6** to determine the important circuit parameters necessary for component selection.*

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The transformer makers' rules don't state where the numbers come from or whether they are applicable to all operating conditions.

across the capacitor) is also negligible. The models don't ignore the internal impedance of the transformer, however, because it's too important.

Although this article uses many first-order approximations to describe power-supply operation, the design rules and graph derivations are accurate models of real power supplies and are much more accurate for a wide range of designs than are the rules of thumb. **Table 1** shows the transformer makers' simple rules of thumb for selecting a transformer for a 1A power supply with capacitive filtering. Depending on whether you're using a half-wave, full-wave bridge, or full-wave center-tap rectifier, you'll need a 2.4, 1.8, or 1.2A transformer. Although the numbers are right, the rules don't state where the numbers come from or whether they are applicable under all operating conditions. In fact, they aren't.

To understand why, you may at this point want to refer to the equations derived in the **box**. A transformer's specified voltage V_S, specified current I_S, and load-regulation factor F_X are all constant characteristics of the transformer. The conduction angle δ, dc output voltage V_O, dc output current I_O, peak diode forward current I_F, rms transformer current I_T, and rms capacitor current I_C are all variables that depend on how much power the supply actually delivers.

TABLE 1—RULES OF THUMB FOR TRANSFORMER SELECTION

TRANSFORMER/RECTIFIER TYPE	FILTER TYPE	REQUIRED RMS SECONDARY RATING
HALF-WAVE	CAPACITOR	2.4×DC CURRENT
FULL-WAVE BRIDGE	CAPACITOR	1.8×DC CURRENT
FULL-WAVE CENTER-TAP	CAPACITOR	1.2×DC CURRENT

TABLE 2 — RULES OF THUMB VERIFIED

TO OBTAIN:	MULTIPLY TRANSFORMER-TYPE FACTOR:			BY:
	HALF-WAVE	FULL-WAVE BRIDGE	FULL-WAVE CENTER-TAP	
F_X	0.90	0.90	0.90	—
I_T	2.39	1.81	1.19	I_O
V_O	1.24	*	0.62	V_S**
I_F	7.16	4.12	3.58	I_O
I_O	1.00	1.00	1.00	I_O
I_C	2.17	1.51	1.31	I_O

*$V_O=(1.32\times V_S)-(2\times V_F)$

**AFTER MULTIPLICATION, SUBTRACT V_F FOR EXACT RESULT.

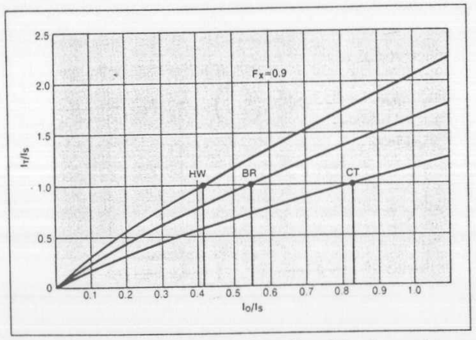

Fig 2—This graph of I_T/I_S vs I_O/I_S shows the points where each curve crosses the $I_T/I_S=1$ line. These points represent the maximum allowable transformer load.

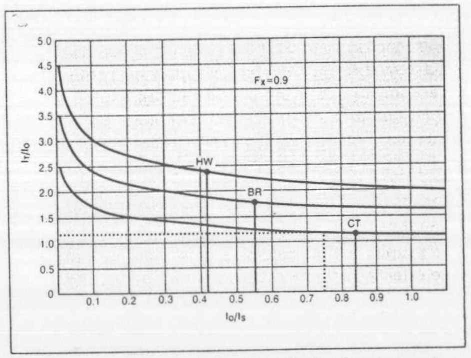

Fig 3—This graph aids in transformer selection. I_S is the transformer maker's maximum specified transformer current.

The maximum allowable power dissipation in the transformer occurs when $I_T=I_S$—when the transformer's rms current under load equals the manufacturer's rated maximum current. Plugging this condition into **Eqs 2, 4,** and **6** in the **box** generates **Table 2**'s list of relationships for a transformer dissipating its maximum allowable power. (**Table 2** expresses current in terms of I_O because engineers generally think of a power supply in terms of its output current.)

Rules verified in one instance

These results verify the transformer makers' rules of thumb: A 1A supply using a half-wave rectifier requires a 2.39A transformer; a 1A supply with a full-wave-bridge rectifier requires a 1.81A transformer; and a 1A supply with a full-wave center-tap rectifier requires a 1.19A transformer. As stated earlier, though, these results are only valid for the transformer under full load and with a load-regulation factor of 0.9.

Fig 2 plots I_T/I_S vs I_O/I_S for the three topologies; HW stands for half-wave, BR stands for full-wave bridge, and CT stands for full-wave center-tap. The graph shows the points where each curve crosses the $I_T/I_S=1$ line. These points represent the maximum allowable transformer load. The X-axis coordinates of these maximum-load points are simply the reciprocals of the 2.39, 1.81, and 1.19 factors in **Table 1**. Operating to the right of these points would overload the transformer.

Figs 3 through **6** are similar graphs; they plot I_T, V_O, I_F, and I_C with respect to I_O/I_S. All the graphs assume that the transformer's load-regulation factor F_X is 0.9. For more precise results, use the exact value of F_X for the transformer you are using and replot the graphs from the equations in the **box**.

The graphs may indicate some unexpected results. A simple example will serve as an illustration. For a 1A power supply with a 10A transformer and a half-wave rectifier, $I_O/I_S=0.1$. The graphs indicate that the capacitor rms current will be 2.875A, the transformer rms current will be 3.05A, and the diode peak forward current will be 11.6A. Assuming a diode forward-voltage drop of 1V, a 10V transformer will provide a dc output voltage of 13.5V.

To fully comprehend how to use the graphs, consider a more realistic example: a 3A, 20V power supply suitable for regulation to 15V. First, you have to decide

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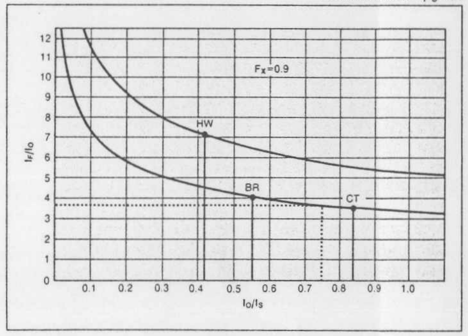

Fig 5—This graph predicts the forward current that your rectifier diode will have to handle.

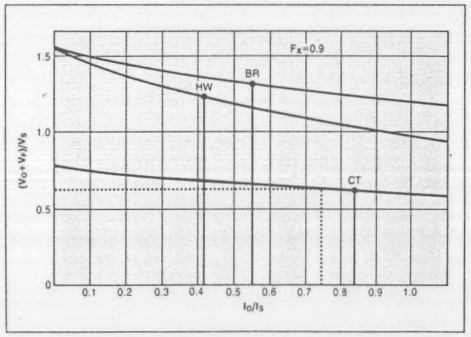

Fig 4—After selecting your transformer, you can use this graph to predict your power supply's output voltage. (V_F is the rectifier's forward-voltage drop).

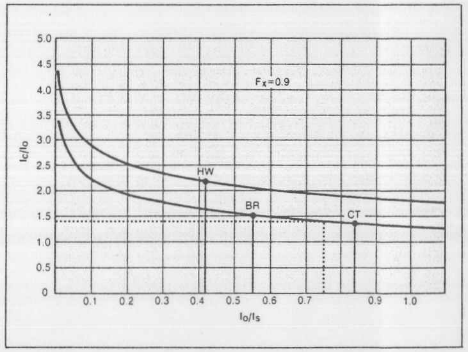

Fig 6—Using this graph will ensure proper sizing of your power supply's output-filter capacitor.

Circuit models yield design equations

To model a real transformer, you can use an ideal voltage source $V_M\sin(t)$ in series with an internal impedance R_X. In the case of a center-tap transformer, half of the voltage and half of the impedance appear on each half of the secondary winding. With the transformer connected to a load, the current flowing through R_X causes a voltage drop across R_X and reduces the transformer's terminal voltage.

Transformer makers specify a transformer's rms voltage (V_S) and rms current (I_S). The ratio of V_S to the open-circuit voltage, typically 0.8 to 0.9, is the transformer's load-regulation factor (F_X).

The transformer equations for F_X, R_X, and P_S (power) are

$$F_X = \frac{\sqrt{2}V_S}{V_M} = \frac{\text{specified rms voltage}}{\text{open-circuit rms voltage}}$$

$$R_X = \frac{(V_M/\sqrt{2})-V_S}{I_S} = \left(\frac{1}{F_X}-1\right)\frac{V_S}{I_S} = \frac{(1-F_X)V_M/\sqrt{2}}{I_S}$$

$$P_S = I_S^2\,R_X = \left(\frac{1}{F_X}-1\right)\times V_S\times I_S.$$

Equivalent circuits

Now consider the equivalent circuit of a simple half-wave power supply (**Fig 1a** in the accompanying article). Engineers often assume that the filter capacitor charges to V_M at the peak of the rectifier output, as **Fig Aa** purports to show. This assumption is invalid, because current flowing through the transformer produces a voltage drop across R_X, which reduces the transformer's terminal voltage. If the transformer's terminal voltage is reduced, the filter capacitor cannot charge to V_M.

In the alternative model in **Fig Ab**, current only flows when the transformer's output voltage exceeds the supply's output voltage (plus the forward-voltage drop of the series diode). Nonetheless, assume that the capacitor is so large that the change in voltage across it during this conduction interval is negligible. Because V_O and V_F are both constants, the transformer's terminal voltage is clamped at V_O+V_F. During the entire time $0<t<2\pi$, a constant current $I_O=V_O/R_L$ flows through the load.

Based on the conduction angle, δ, and the transformer's V_S, I_S, and F_X, you can calculate the following circuit parameters: the dc filter output voltage (V_O), the dc filter output current (I_O), the peak diode forward current (I_F), the rms transformer current (I_T), and the rms filter capacitor current (I_C). You can generally read the peak diode forward voltage V_F from the diode's data sheet if you know I_F.

First, the transformer voltage at which the rectifier begins to conduct is

$$V_M\sin(\delta) = V_O+V_F.$$

Or, in terms of the dc filter output voltage,

$$V_O = \frac{\sqrt{2}\sin(\delta)}{F_X}V_S-V_F.$$

The peak diode current occurs when the voltage across the transformer's internal impedance is at its maximum—which equals the maximum sinewave voltage minus the transformer's terminal voltage:

$$I_F = \frac{V_M-(V_O+V_F)}{R_X} = \frac{\sqrt{2}[1-\sin(\delta)]}{1-F_X}\times I_S. \quad (1)$$

The instantaneous transformer current, I_X, during conduction is

$$I_X = \frac{V_X}{R_X} = \frac{V_M\sin(t)-(V_O+V_F)}{R_X}.$$

Integrating the instantaneous current and dividing by the period yields the average transformer current:

$$\text{average current} = \frac{1}{T}\int_0^T I_X\times dt = \frac{1}{2\pi}\int_\delta^{\pi-\delta}\frac{V_M\sin(t)-(V_O+V_F)}{R_X}\times dt.$$

Because the average voltage across the capacitor is constant, the average current through the capacitor must be zero. Therefore, the average transformer current must be equal to I_O. Solving for this equation yields

$$I_O = \frac{2\cos(\delta)+(2\delta-\pi)\sin(\delta)}{\pi(1-F_X)\sqrt{2}}\times I_S.$$

Plugging the instantaneous current into the standard rms integral equation gives

$$\text{rms current} = \sqrt{\frac{1}{T}\int_0^T I_X^2\times dt} = \sqrt{\frac{1}{2\pi}\int_\delta^{\pi-\delta}\left(\frac{V_M\sin(t)-(V_O+V_F)}{R_X}\right)^2\times dt},$$

which yields

$$I_T=\frac{1}{1-F_X}\sqrt{\frac{1}{\pi}[(\pi-2\delta)[\tfrac{1}{2}+\sin^2(\delta)]-\tfrac{3}{2}\sin(2\delta)]}\times I_s. \quad (2)$$

Although the average current through the filter capacitor is zero, the capacitor does charge and discharge. Its rms current is

$$I_C = \sqrt{I_T^2-I_O^2}.$$

The equations for full-wave bridge and full-wave center-tap rectifier circuits are simple extensions of the half-wave rectifier equations. Consider the full-wave bridge power-supply equivalent circuit first (**Fig 1b** in the accompanying article). There are two differences between the full-wave bridge and the half-wave circuits:

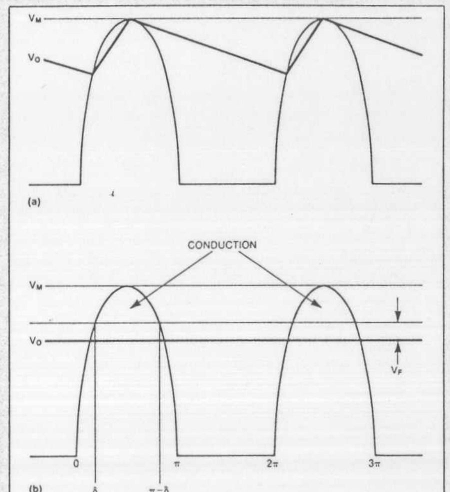

Fig A—You are incorrect if you assume that the filter capacitor charges to V_M at the peak of the rectifier output as Aa shows. Ab's correct model of filter-capacitor operation shows current flowing only when the transformer's output voltage exceeds the supply's output voltage (plus the forward-voltage drop of the series diode).

The full-wave bridge supply can have two diode forward-voltage drops at any time, and the period of the transformer current is π instead of 2π.

These differences result in the following equations for the full-wave bridge rectifier circuit:

$$V_M\sin(\delta) = V_O+2V_F$$

$$V_O = \frac{\sqrt{2}\sin(\delta)}{F_X}V_S-2V_F$$

$$I_F = \frac{\sqrt{2}[1-\sin(\delta)]}{1-F_X}\times I_S \quad (3)$$

$$I_O = \frac{\sqrt{2}[2\cos(\delta) + (2\delta-\pi)\sin(\delta)]}{\pi(1-F_X)}\times I_S$$

$$I_T = \frac{1}{1-F_X}\sqrt{\frac{2}{\pi}[(\pi-2\delta)[\tfrac{1}{2}+\sin^2(\delta)]-\tfrac{3}{2}\sin(2\delta)]}\times I_S \quad (4)$$

$$I_C = \sqrt{I_T^2-I_O^2}.$$

Next, consider the equivalent circuit for a full-wave center-tap power supply (**Fig 1c** in the accompanying article). There are four differences between the full-wave center-tap and half-wave circuits: The peak transformer voltage is $V_M/2$, the transformer impedance in each leg is $R_X/2$, the period of the current charging the capacitor is π instead of 2π, and I_T is defined as the current flowing through one leg of the transformer, resulting in two paths of current through the rectifier diodes to the filter capacitor.

These differences result in the following equations for the full-wave center-tap rectifier circuit:

$$V_M\sin(\delta)/2 = V_O+V_F$$

$$V_O = \frac{\sin(\delta)}{F_X\sqrt{2}}V_S-V_F$$

$$I_F = \frac{\sqrt{2}[1-\sin(\delta)]}{1-F_X}\times I_S \quad (5)$$

$$I_O = \frac{\sqrt{2}[2\cos(\delta)+(2\delta-\pi)\sin(\delta)]}{\pi(1-F_X)}\times I_S$$

$$I_T = \frac{1}{1-F_X}\sqrt{\frac{1}{\pi}[(\pi-2\delta)[\tfrac{1}{2}+\sin^2(\delta)]-\tfrac{3}{2}\sin(2\delta)]}\times I_S \quad (6)$$

$$I_C = \sqrt{2I_T^2-I_O^2}.$$

which topology to use. Supposing you use a full-wave bridge rectifier, you can see by looking at **Fig 3** that you need at least a 5.4A transformer. **Fig 4** indicates that the transformer should be rated at about 16.7V (assuming 1V diode forward-voltage drops). If you use a full-wave center-tap rectifier, you need a 3.6A, 33.9V transformer.

In this case, the center-tap rectifier circuit is the topology of choice because of the availability of a stock 4A, 36V transformer (Stancor P-8673). Going back to the graphs armed with this transformer's parameters, you can see that $I_O/I_S=0.75$ (indicated by a dotted line in Fig 3). **Fig 3** also indicates that the transformer rms current will be 3.7A. **Fig 4** predicts a dc output voltage of 21.8V, resulting in the voltage regulator dissipating 20.4W. **Fig 5** shows that the diodes must be rated for a repetitive peak forward current of 11.1A, and **Fig 6** indicates that the filter capacitor must be able to withstand an rms current of 4.2A.

You should be aware of one other salient parameter when choosing a transformer. When the power supply is first turned on, the voltage across the filter capacitor is zero, momentarily short-circuiting the transformer. This short circuit causes the entire peak voltage of the transformer to be dropped across the transformer's internal resistance because of the large current flowing through the rectifier into the capacitor's effective ground. The rectifier diodes must be able to withstand this momentary surge of current (diode manufacturers specify it as I_{FSM}). Using **Eqs 1, 3,** and **5** from the **box,** you can calculate I_{FSM} for a half-wave, full-wave bridge, and full-wave center-tap circuit, respectively. For the example in the previous paragraph, $I_{FSM}=56.6A$. **EDN**

Author's biography

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Article Interest Quotient (Circle One)
High 485 Medium 486 Low 487